

# Classical-driving-assisted quantum speed-up

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We propose a method of accelerating the speed of evolution of an open system by an external classical driving field for a qubit in a zero-temperature structured reservoir. It is shown that, with a judicious choice of the driving strength of the applied classical field, a speed-up evolution of an open system can be achieved in both the weak system-environment couplings and the strong system-environment couplings. By considering the relationship between non-Markovianity of environment and the classical field, we can drive the open system from the Markovian to the non-Markovian regime by manipulating the driving strength of classical field. That is the intrinsic physical reason that the classical field may induce the speed-up process. In addition, the roles of this classical field on the variation of quantum evolution speed in the whole decoherence process is discussed.

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*Introduction.*—In virtually all areas of quantum physics a fundamental and important task arising is to drive a given initial state to a target state in the minimum evolution time. This problem involves in many areas of research such as quantum communication [1, 2], quantum metrology [3], quantum computation [4], nonequilibrium thermodynamics [5], as well as quantum optimal control protocols [6–10]. The minimum evolution time between two distinguishable states of a system, which be defined as quantum speed limit time (QSLT) [11–27], is a key method in characterizing the maximal speed of evolution of quantum systems. Since the relevant influence of the environment on processing or information transferring systems can not be ignored, the unified bounds of evolution time including both Mandelstam-Tamm (MT) and Margolus-Levitin (ML) types focused on the open system with nonunitary dynamics process have been formulated [25, 26]. Interestingly, the QSLT would equal to the actual driving time in the weak system-environment couplings, while the strong system-environment couplings can reduce the QSLT below the actual driving time [25]. This fact means that the strong system-environment couplings can speed up the quantum evolution process. However, under the weak system-environment couplings, the accelerating of quantum evolution is generally not achieved without any operating to the system. And as we all know that a speed-up evolution of an open system would be preferable to deal with the robustness of quantum simulators and computers against decoherence [28, 29]. So how to devise an effective and feasible mechanism to speed up the evolution process of an open system under more general physical conditions such as in weak-coupling case, becomes extremely significant.

In this Letter, we will investigate a generic decoherence model of a qubit interacting with a zero-temperature

structured reservoir and driven by an external classical field. We demonstrate how a speed-up evolution of an open system can be acquired by manipulating the driving strength of the classical field, although the system-environment coupling is weak. By investigating the influence of the classical field on the QSLT, for a certain critical driving strength of the classical field, a sudden transition from no speed-up to speed-up can occur in the weak-coupling regime. Additionally under the strong-coupling regime, the speed of evolution for the system can also be controlled to a speed-up or speed-down process by the appropriate driving strength of the classical field.

According to Ref. [25], the speed-up evolution of the open system is mainly related to the non-Markovianity of the environment [30–39]. So in order to clear the physical reason of the speed-up process induced by the classical field, we further focus on the relationship between the non-Markovianity of environment and the classical field. We note that the original Markovian dynamics can be changed to the non-Markovian dynamics by choosing an agreeable driving strength of the classical field. And the transition point from Markovian dynamics to non-Markovian dynamics is interestingly equal to the critical driving strength where the uniform evolution speed becomes the speed-up dynamical process of the open system. Finally, we explore the effects of the classical field on the variation of quantum evolution speed in the whole decoherence process by calculating the QSLT for the arbitrary time-evolution state. Remarkably, the applied classical field can result in the smaller acceleration in the speed-up process and the smaller deceleration in the speed-down process.

*Model.*—Here, we consider a two-level system interacting with a structured reservoir at zero temperature. A

specific system which consists of a two-level atom (transition frequency  $\omega_0$ ) interacting with a electromagnetic field has been chosen in this Letter. And the atom is driven by a classical field with frequency  $\omega_L$ . The Hamiltonian reads,

$$H = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k a_k^\dagger a_k + \Omega(e^{-i\omega_L t}\sigma_+ + e^{i\omega_L t}\sigma_-) + \sum_k g_k(a_k\sigma_+ + a_k^\dagger\sigma_-), \quad (1)$$

where the operators  $\sigma_z$  and  $\sigma_\pm$  are defined by  $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ ,  $\sigma_+ = |e\rangle\langle g|$ , and  $\sigma_- = \sigma_+^\dagger$  associated with the upper level  $|e\rangle$  and the lower level  $|g\rangle$ ;  $a_k$  and  $a_k^\dagger$  are the annihilation and creation operators for the field mode  $k$ , which is characterized by the frequency  $\omega_k$ ;  $g_k$  and  $\Omega$ , both chosen to be real, are the coupling constants of the interactions of the atom with the field mode  $k$  and with the classical driving field, respectively. In the dressed-state basis  $\{|+\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)\}$ , by considering two rotating reference frame through two unitary transformation  $U_1 = \exp[-i\omega_L\sigma_z t/2]$  and  $U_2 = \exp[i\omega_0\Sigma_z t/2]$  [40, 41], the total Hamiltonian in Eq. (1) can be transferred to an effective Hamiltonian in the rotating-wave approximation,

$$H_{eff} = \frac{\omega'}{2}\Sigma_z + \sum_k \omega_k a_k^\dagger a_k + \sum_k g'_k[a_k\Sigma_+ + a_k^\dagger\Sigma_-], \quad (2)$$

with  $\omega' = 2\Omega + \omega_0$  and  $g'_k = g_k/2$ . Here  $\Sigma_z$  and  $\Sigma_\pm$  are defined by  $\Sigma_z = |+\rangle\langle +| - |-\rangle\langle -|$ ,  $\Sigma_+ = |+\rangle\langle -|$ , and  $\Sigma_- = \Sigma_+^\dagger$ . A noteworthy feature of this effective Hamiltonian is that the basis states have been changed to  $\{|+\rangle, |-\rangle\}$ , when the atom coupled with the structured reservoir with the assistance of the external classical field.

Furthermore, at zero temperature, let us consider the situation where the initial state of the system plus reservoir is of the form  $|\Psi(0)\rangle = |+\rangle_S|\mathbf{0}\rangle_E$ , with  $|\mathbf{0}\rangle_E$  denotes the vacuum state of the reservoir. By the Hamiltonian described in Eq. (2), the state of the total system at any time  $t$  is given by,  $|\Psi(t)\rangle = c_+(t)|+\rangle_S|\mathbf{0}\rangle_E + \sum_k c_k(t)|-\rangle_S|1_k\rangle_E$ , where  $|1_k\rangle_E$  is the state of the reservoir with only one excitation in the  $k$ -th mode. The time evolution of the probability amplitudes is governed by a series of differential equations,

$$\dot{c}_+(t) = -i \sum_k g'_k \exp[i(\omega' - \omega_k)t] c_k(t), \quad (3)$$

$$\dot{c}_k(t) = -i g'_k \exp[-i(\omega' - \omega_k)t] c_+(t). \quad (4)$$

Owing to no excitations in the initial state of the reservoir, that is  $c_k(0) = 0$ , we can obtain the integro-differential equation for  $c_+(t)$  as  $\dot{c}_+(t) = -\int_0^t dt_1 f(t - t_1) c_+(t_1)$ . The correlation function  $f(t - t_1)$  is related to the spectral density  $S(\omega)$  of the reservoir. Here, the environment can be

described by an effective Lorentzian spectral density of the form  $S(\omega) = \frac{1}{2\pi} \frac{\lambda R}{(\omega - \omega_c)^2 + \lambda^2}$ , where  $\lambda$  is the spectral width,  $R$  the coupling strength, and  $\omega_c$  in the center frequency of the reservoir. Typically, in weak-coupling regime ( $\lambda > 2R$ ), the behavior of the qubit-cavity system is Markovian and irreversible decay occurs. For strong-coupling regime ( $\lambda < 2R$ ), non-Markovian dynamics occurs accompanied by an oscillatory reversible decay. Through introducing the correlation function  $f(t - t_1) = \int d\omega S(\omega) e^{i(\omega' - \omega)(t - t_1)}$  and performing the Laplace transform, we acquire  $s\tilde{c}_+(s) - c_+(0) = -\tilde{c}_+(s)f(s)$ . From the above equation we can derive the quantity  $\tilde{c}_+(s)$ . Finally, inverting the Laplace transform we can obtain a formal solution for the amplitude  $c_+(t) = \varepsilon(t)c_+(0)$ , with  $\varepsilon(t) = e^{-[\lambda - i(\omega' - \omega_c)]t/2} [\cosh(Dt/2) + \frac{\lambda - i(\omega' - \omega_c)}{D} \sinh(Dt/2)]$ , where  $D = \sqrt{\lambda^2 - 2R\lambda - (\omega' - \omega_c)^2 - 2i(\omega' - \omega_c)\lambda}$ . In the dressed-state basis, the reduced density matrix of the system at time  $t$  reads,

$$\rho_t = \begin{pmatrix} \rho_{++}(0)|\varepsilon(t)|^2 & \rho_{+-}(0)\varepsilon(t) \\ \rho_{-+}(0)\varepsilon^*(t) & 1 - \rho_{++}(0)|\varepsilon(t)|^2 \end{pmatrix}. \quad (5)$$

*Speed-up of quantum evolution from  $\rho_0$  to  $\rho_{\tau_D}$ .*—In order to illustrate the role of the external classical field on the quantum speed of evolution of the open system, we firstly start with the definition of the QSLT for open quantum system. the QSLT can effectually define the bound of minimal evolution time for arbitrary initial states, and be helpful to analyze the maximal speed of evolution of open quantum system. A unified lower bound, including both MT and ML types, has been derived by Deffner and Lutz [25]. The QSLT is determined by an initial state  $\rho_0 = |\phi_0\rangle\langle\phi_0|$  and its target state  $\rho_{\tau_D}$ , governed by the master equation  $\dot{\rho}_t = L_t\rho_t$ , with  $L_t$  the positive generator of the dynamical semigroup. With the help of the von Neumann trace inequality and the Cauchy-Schwarz inequality, the QSLT is as follows,

$$\tau_D \geq \tau_{QSL} = \max\left\{\frac{1}{\Lambda_{\tau_D}^1}, \frac{1}{\Lambda_{\tau_D}^2}, \frac{1}{\Lambda_{\tau_D}^\infty}\right\} \sin^2[\mathfrak{B}(\rho_0, \rho_{\tau_D})], \quad (6)$$

with  $\Lambda_{\tau_D}^p = \tau_D^{-1} \int_0^{\tau_D} \|L_t\rho_t\|_p dt$ , and  $\|A\| = (\sigma_1^p + \dots + \sigma_n^p)^{1/p}$  denotes the Schatten  $p$  norm,  $\sigma_1, \dots, \sigma_n$  are the singular values of  $A$ ,  $\mathfrak{B}(\rho_0, \rho_{\tau_D}) = \arccos \sqrt{\langle\phi_0|\rho_{\tau_D}|\phi_0\rangle}$  denotes the Bures angle between the initial and target states of the quantum system. And the ML-type bound based on the operator norm ( $p = \infty$ ) of the nonunitary generator provides the sharpest bound on the QSLT [25]. So in the following we will use this ML-type bound to demonstrate the speed of the dynamics evolution from an initial state  $\rho_0$  to a final state  $\rho_{\tau_D}$  by a driving time  $\tau_D$ .

We shall examine the dynamics process where the system starts in the dressed state  $|+\rangle$ , that is  $\rho_{++}(0) = 1$  and  $\rho_{+-}(0) = 0$ . If there is no classical field to drive

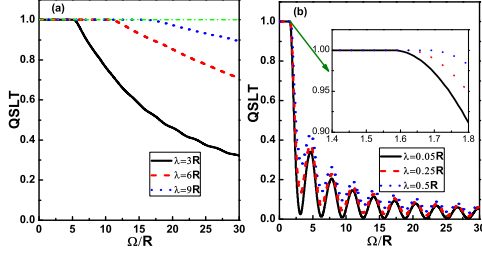


FIG. 1: (Color online) The QSLT for an open system driven by an external classical field as a function of the parameter variable  $\Omega/R$ , (a) in the weak-coupling regime ( $\lambda > 2R$ ), and (b) in the strong-coupling regime ( $\lambda < 2R$ ). The green dash-dotted line in (a) represents the actual driving time  $\tau_D = 1$ .

the system ( $\Omega = 0$ ), and the system is in resonance with the center mode of the reservoir, i.e.  $\omega_0 = \omega_c$ , the QSLT would equal to the actual driving time  $\tau_D$  in the weak-coupling regime ( $\lambda > 2R$ ), that is no speed-up dynamics process [25]. In order to obtain the quantum speed-up of the evolution process in the weak-coupling regime, we show how to manipulate the QSLT for the open system via a classical field. Fig. 1(a) shows the QSLT for an open system as a function of the driving strength of the external classical field  $\Omega$  in the weak-coupling regime, with the resonance case  $\omega_0 = \omega_c$ , and the actual driving time  $\tau_D = 1$ . It is worth noting that, a remarkable behavior of sudden transition from no speed-up to speed-up can occur at a certain critical driving strength of the classical field  $\Omega_c$ . When  $\Omega < \Omega_c$ , the QSLT of the system is actually the driving time, and then decreases monotonically with increasing  $\Omega$ . So we therefore reach the interesting result that the external classical field can be used to reduced the QSLT below its value in the weak-coupling regime. Thus we obtain the speed-up of the evolution of an open quantum system in the weak-coupling regime. And then, numerical calculation also shows that the critical driving strength  $\Omega_c$  is determined by the value of the spectral width  $\lambda$ . The larger the value of  $\lambda$  is, the larger the value of the critical driving strength  $\Omega_c$  should be requested. Take the cases in Fig. 1(a) as examples, when  $\lambda = 3R$ , we find the value of the critical driving strength is  $\Omega_c = 5.31R$ . While in the cases  $\lambda = 6R$  and  $\lambda = 9R$ , we can acquire  $\Omega_c = 10.89R$  and  $\Omega_c = 16.41R$ , respectively.

Moreover, for the strong-coupling regime ( $\lambda < 2R$ ), the QSLT exhibits a plateau independent of  $\Omega$  for the moderate driving strength of the classical field, and then periodically decrease for the large driving strength of the classical field (as shown in Fig. 1(b)). That is to say, in the strong-coupling regime, the speed of evolution for the system can be controlled to a speed-up or speed-down process by the driving strength  $\Omega$ .

As noted in Refs. [25–27], the non-Markovianity in the dynamics process ( $\rho_0$  to  $\rho_{\tau_D}$ ), and the associated in-

formation backflow from the reservoir, can lead to faster quantum evolution, and hence to smaller QSLT. In order to understand the physical reason of the speed-up process, in what follows we would describe a scheme how to turn the dynamics from Markovian to non-Markovian by adding an external classical field to the qubit. The measure  $\mathbb{N}(\Phi)$  for non-Markovianity of the

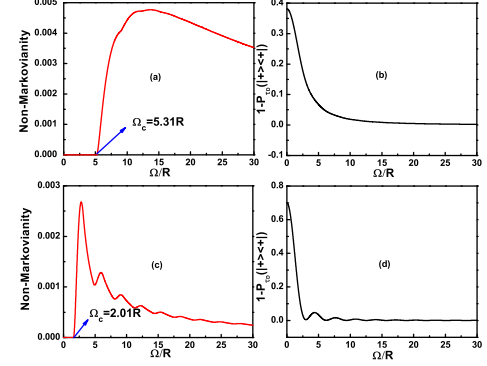


FIG. 2: (Color online) The non-Markovianity  $\mathbb{N}(\Phi)$  (red solid line) and the population  $1 - P_{\tau_D}(|+\rangle\langle+|)$  (black solid line) for an open system driven by an external classical field as a function of the parameter variable  $\Omega/R$ , with  $\tau_D = 1$ . (a) and (b) in the weak-coupling regime,  $\lambda = 3R$ , (c) and (d) in the strong-coupling regime,  $\lambda = 0.05R$ .

quantum process  $\Phi(t)$  has been defined by Breuer *et al.* [31]. Considering a quantum process  $\Phi(t)$ ,  $\rho(t) = \Phi(t)\rho(0)$ , where  $\rho(0)$  and  $\rho(t)$  denote the density operators at time  $t = 0$  and at any time  $t > 0$ , respectively, then the non-Markovianity  $\mathbb{N}(\Phi)$  is defined as  $\mathbb{N}(\Phi) = \max_{\rho_{1,2}(0)} \int_{\sigma > 0} dt \sigma(t, \rho_{1,2}(0))$ , where  $\sigma(t, \rho_{1,2}(0))$  is the rate of change of the trace distance,  $\sigma(t, \rho_{1,2}(0)) = \frac{d}{dt} \mathcal{D}(\rho_1(t), \rho_2(t))$ . The trace distance  $\mathcal{D}$  describing the distinguishability between the two states is defined as [1]  $\mathcal{D}(\rho_1, \rho_2) = \frac{1}{2} \|\rho_1 - \rho_2\|$ , where  $\|M\| = \sqrt{M^\dagger M}$  and  $0 \leq \mathcal{D} \leq 1$ . And  $\sigma(t, \rho_{1,2}(0)) \leq 0$  corresponds to all dynamical semigroups and all time-dependent Markovian processes, a process is non-Markovian if there exists a pair of initial states and at certain time  $t$  such that  $\sigma(t, \rho_{1,2}(0)) > 0$ . We should take the maximum over all initial states  $\rho_{1,2}(0)$  to calculate the degree of non-Markovianity. Similar to Refs. [31, 33], by drawing a sufficiently large sample of random pairs of initial states, the optimal state pair is attained for the initial states  $\rho_-(0) = |-\rangle\langle-|$  and  $\rho_+(0) = |+\rangle\langle+|$  by strong numerical calculations in the dressed-state basis  $\{|-\rangle, |+\rangle\}$ . Here, for the optimal state pair, the rate of change of the trace distance can be acquired  $\sigma(t, \rho_{+,-}(0)) = \partial_t |\varepsilon(t)|^2$ , and the singular values of the nonunitary generator  $L_t \rho_t$  are given by  $|\sigma(t, \rho_{+,-}(0))|$ .

In the light of Eq. (6), the QSLT for the system can be clearly derived as  $\tau_{QSL} = \frac{\tau_D [1 - P_{\tau_D}(|+\rangle\langle+|)]}{\int_0^{\tau_D} |\sigma(t, \rho_{+,-}(0))| dt} = \frac{\tau_D [1 - P_{\tau_D}(|+\rangle\langle+|)]}{2\mathbb{N}(\Phi) + 1 - P_{\tau_D}(|+\rangle\langle+|)}$ , where  $P_{\tau_D}(|+\rangle\langle+|) = |\varepsilon(\tau_D)|^2$  is

the population of the dressed-state  $|+\rangle$  at time  $\tau_D$ . It is easy to find that the QSLT is strictly related to the non-Markovianity of the evolution from  $\rho_0$  to  $\rho_{\tau_D}$  and the population of the dressed-state  $|+\rangle$  at time  $\tau_D$ . Then we investigate the effects of the driving classical field on the non-Markovianity  $N(\Phi)$  and the population  $1 - P_{\tau_D}(|+\rangle\langle+|)$ , as shown in Fig. 2. Both in the weak-coupling regime ( $\lambda = 3R$ ) and in the strong-coupling regime ( $\lambda = 0.05R$ ), by considering the evolution process within the driving time, Figs. 2(a) and 2(c) illustrate that the original Markovian dynamics can be changed to the non-Markovian dynamics by choosing an agreeable driving strength of the classical field. And the transition point from Markovian dynamics to non-Markovian dynamics is equal to the critical driving strength of the classical field  $\Omega_c$  where the uniform evolution speed becomes the speed-up dynamical process of the system. On the other hand, a nonmonotonic behavior of the non-Markovianity can be shown in Fig. 2: when  $\Omega > \Omega_c$ , the non-Markovianity firstly increase with increasing  $\Omega$ , after it reaches a maximum value, it decreases with further increasing of  $\Omega$ . However, Figs. 2(b) and 2(d) show that the population  $1 - P_{\tau_D}(|+\rangle\langle+|)$  converges to zero for the strong driving classical field in the weak-coupling regime (no oscillations) and the strong-coupling regime (oscillations are present). So the QSLT can still be reduced by the classical field, that means the external classical field can be used to control the speed of evolution of quantum systems.

*Variation of quantum evolution speed of the whole decoherence process.*—One may naturally concern the variation of a speed for QSLT based on an arbitrary time-evolution state  $\rho_\tau$ . The QSLT for mixed initial states [26] can be used to demonstrate the quantum speed of evolution from  $\rho_\tau$  to another  $\rho_{\tau+\tau_D}$  by a driving time  $\tau_D$ . Here, we mainly examine the whole dynamics process where the system starts from the dressed state  $|+\rangle$  in the weak-coupling regime. By calculating the singular values of  $\rho_\tau$  and  $L_t\rho_t$ , the singular values for  $\rho_\tau$  are  $\varrho_1 = P_\tau(|+\rangle\langle+|)$  and  $\varrho_2 = 1 - P_\tau(|+\rangle\langle+|)$ , while for  $L_t\rho_t$ , the singular values are  $\sigma_1 = \sigma_2 = |\dot{P}_t(|+\rangle\langle+|)|$ . Then the QSLT for a time-evolution state  $\rho_\tau$  can be calculated  $\tau_{QSL} = \frac{\tau_D[1-2P_\tau(|+\rangle\langle+|)][P_\tau(|+\rangle\langle+|)-P_{\tau+\tau_D}(|+\rangle\langle+|)]}{\int_{\tau}^{\tau+\tau_D} |\dot{P}_t(|+\rangle\langle+|)|dt}$ . Figs. 3(a) and 3(b) present the results of our analysis for  $\tau_{QSL}$  by choosing different driving strength  $\Omega$  of the applied classical field, with  $\lambda = 3R$ . By adding an external classical field to the system, we observe that, the evolution of the open system can first execute a speed-up process and then show gradual deceleration process in the case  $\Omega < \Omega_c = 5.31R$ . The case  $\Omega > \Omega_c = 5.31R$  is complicated which can be explained by non-Markovianity, see Fig. 3(b). Overall, a remarkable result we find that, for the speed-up process, the decay rate of the QSLT can decrease with the driving strength  $\Omega$  increasing. And for the speed-down process, the increasing rate of the QSLT

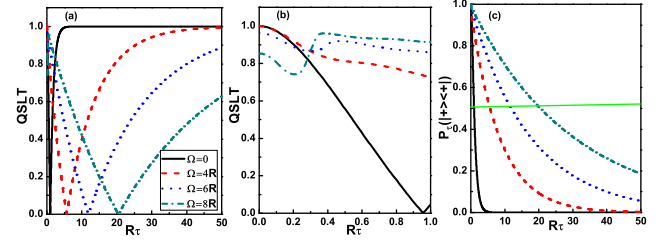


FIG. 3: (Color online) The QSLTs (a), (b) and the population  $P_\tau(|+\rangle\langle+|)$  (c) for the arbitrary time-evolution state as a function of the initial time parameter  $R\tau$  by different driving strength  $\Omega$  of the applied classical field. Here we consider an open dynamics process started from the dressed state  $|+\rangle$ , and parameters are chosen as  $\lambda = 3R$ ,  $\tau_D = 1$ . The green dash-dot-dotted line in (c) represents the population  $P_{\tau_c}(|+\rangle\langle+|) = 0.5$ , which can be used to determine the critical time point between the speeded-up process and the speed-down process.

would also be reduced by choosing a stronger driving classical field. This can be understood that the applied classical field can lead to the smaller acceleration in the speed-up process, and also the smaller deceleration in the speed-down process. This is a newly noticed phenomenon. Finally, as shown in Fig. 3(c), the increasing of the driving strength  $\Omega$  makes the energy exchange between the system and the environment more slow. This behavior plays the dominating role on the variation of quantum evolution speed in the whole decoherence process.

*Conclusion.*—In summary, we demonstrated that a speed-up evolution of an open system could be achieved by manipulating the driving strength of an external classical field. We show that the phenomenon of transition from Markovian to non-Markovian dynamics induced by the classical field is the main physical reason of the speed-up process. Recent experiments with photons allow one to drive the open system from Markovian to non-Markovian regime in the dephasing channels [38, 39]. In comparison, the results we illustrated here involve the amplitude damping channels for the controlling of Markovian and non-Markovian dynamics. The potential candidates which can realize this type of quantum state and the environment can be systems such as cavity QED [42], trapped ions [43], superconducting qubits [44] and the Nitrogen-Vacancy center of diamond [45].

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